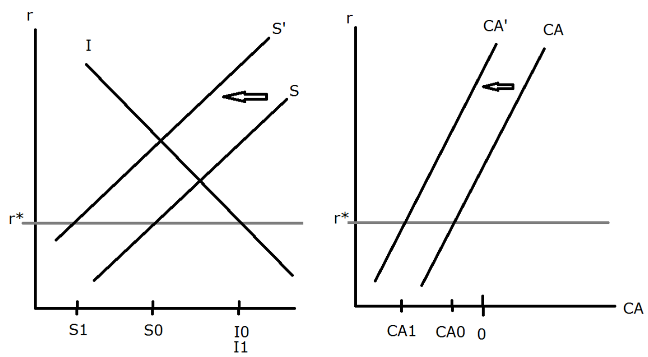
This exam consists of 2 questions. Please answer both.

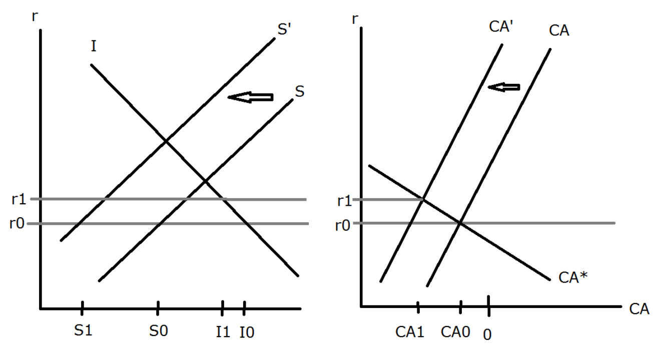
1. Are the following statements true, false or neither, and why? Explain your answer in detail. You do not need to use math for these questions but should feel free to if it helps you be precise.

   (a) Savings and investment are surprisingly highly correlated.

Solution: There is more than one way to answer this question and receive full points. The following is one suggestion. Depends. Feldstein and Horioka compute time-averages of savings and investment rates and compare how correlated they are for a range of OECD countries, and find that they are highly correlated. If savings and investment are highly correlated, that means that the current account is relatively close to zero. This is puzzling when viewed through the lens of a standard small open economy model in which agents should use the current account to smooth consumption intertemporally and to take advantage of investment opportunities by borrowing from abroad. Suppose for example demand for savings increases for some reason. This does not affect the interest rate in the SOE, and thus does not affect investment, so on average savings and investment will be uncorrelated - see the graphical illustration below.



It is less puzzling, for example, when we think of a large open economy where interest rates are endogenous. Consider the increased savings demand example again. In the large open economy, this endogenously increases the interest rate to maintain balance of payment equilibrium globally and thus increases investment - an endogenously positive relationship between the two.



It is of course also less surprising in a closed economy where savings=investment by the national accounts.

(b) The Backus Smith puzzle is that relative consumption levels across countries are not constant.

Solution: There is more than one way to answer this question and receive full points. The following is one suggestion. False. The Backus Smith puzzle refers to the fact that in the data, countries that enjoy relative high consumption levels tend to see their exchange rates appreciate, while a standard model predicts the opposite under complete risk sharing. Intuitively, the model predicts that relative price movements should offset wealth effects at home to facilitate efficient risk sharing across countries.

One can show this by setting up the basic model and deriving the international risk sharing condition that relates the ratio of marginal utilities to the real exchange rate as follows - note that this is not necessary to receive full credit if an equivalent clear verbal explanation is given:

Consider an economy with 2 countries and 2 goods, a and b. Country 1 is endowed with $\bar{A}$, country 2 is endowed with $\bar{B}$. Consumers in both countries consume a composite final good:

$$\begin{aligned} c_1 &= G_1(a_1, b_1) = \left[\omega a_1^{\frac{\sigma-1}{\sigma}} + (1-\omega) b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ c_2 &= G_2(a_2, b_2) = \left[(1-\omega) a_2^{\frac{\sigma-1}{\sigma}} + \omega b_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

with σ elasticity of substitution between a and b and $\omega > 0.5$ determines the degree of home bias in consumption. The social planner problem (SPP) in this economy is

$$\max_{a_j, b_j} \sum_{j=1}^{2} u\left(G_j(a_j, b_j)\right) + \lambda_1(\bar{A} - a_1 - a_2) + \lambda_2(\bar{B} - b_1 - b_2)$$

Define q_j^a and q_j^b as the relative price of a and b, respectively, in terms of the final consumption good in country j, and note that

$$\begin{aligned} q_j^b &= \frac{\partial G_j}{\partial b_j} \\ q_j^a &= \frac{\partial G_j}{\partial a_j} \end{aligned}$$

Then the FOCs to the SPP can be written as

$$\begin{aligned} u'(c_1)q_1^a &= \lambda_1 \\ u'(c_1)q_1^b &= \lambda_2 \\ u'(c_2)q_2^a &= \lambda_1 \\ u'(c_2)q_2^b &= \lambda_2 \end{aligned}$$

Define the real exchange rate rx_t as the price of consumption in country 2 relative to country 1, then the FOCs imply

$$rx \equiv \frac{q_1^a}{q_2^a} = \frac{u'(c_2)}{u'(c_1)}$$

With CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$,

$$rx = \left(\frac{c_2}{c_1}\right)^{-\gamma}$$

Note that this implies

$$\Delta \log(rx) = \gamma \Delta \log(c_1/c_2)$$

This fails to hold in the data - the Backus Smith puzzle. Finally note that in a one good model, the real exchange rate is constant and equal to 1, and thus under perfect risk sharing equal to the relative marginal utilities of consumption - this is also counterfactual, but not what Backus Smith focus on.

2. Consider a two-period small open endowment economy with default. The representative country prefers more consumption c_t to less, and receives an endowment y_t of the consumption good in each period $t = 1, 2$.

   The endowment is exogenous, stochastic and i.i.d. with cumulative distribution function denoted by $F(\cdot)$. If the country is in default, it pays an output cost such that its endowment in default is $y^d = 0.5y$.

   One period bonds b_t that pay one unit of consumption in period t are the only available asset. The country can borrow from international investors in period 1 by issuing debt $b_2 < 0$ at price q. This price is determined in equilibrium. Notice that $b_3 = 0$. Assume that the country inherits no debt, $b_1 = 0$, and that it cannot accumulate assets, $b_t < 0, \forall t$.

International investors are risk-neutral, competitive, maximize expected profits and have cost of funds given by the fixed, exogenous gross international risk-free rate $1+r$.

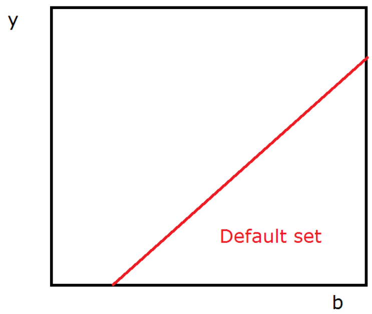
The timing is as follows. In period 1, after receiving y_1, the country decides whether to default or repay. If it repays, it chooses b_2. If it defaults, it pays the output cost. Consumption c_1 takes place. In period 2, after receiving the endowment y_2, if it did not default in period 1, it decides whether to default or repay. If it did default in period 1, it remains in default and pays the output cost. Consumption c_2 takes place.

(a) Solve for the default decision of the country in the final period $t = 2$ as a function of the endowment y_2 and the debt level b_2. Comment on the economics, what determines default decisions in this model?

Solution: The country defaults in period 2 if that yields higher consumption than repaying, i.e. if

$$\begin{aligned} y^d = y_2/2 &> y_2 + b_2 \\ y_2 &< -2b_2 \end{aligned}$$

The state space can thus be split into a default and repayment region - default incentives are higher the higher debt and the lower output is.



(b) Show that the price at which the country can sell its bonds in the first period is given by

$$q(b_2) = \frac{1 - F(y^*(b_2))}{1+r}$$

where $y^*(b_2) = -2b_2$.

Solution: Bond prices are determined by risk-neutral competitive investors maximizing expected profits that choose how much to lend:

$$\max_{b'} \pi = E\left[\frac{(1-d')b'}{1+r} - qb'\right]$$

They take as given default decisions and prices. They lend out qb' and receive (discounted by their cost of funds) $\frac{b'}{1+r}$ back the next period if the country does not default ($d' = 0$), and zero otherwise. The associated first order condition is

$$q = \frac{E[1-d']}{1+r} \tag{1}$$

An alternative approach instead of writing down the profit maximization problem is the following. The expected return on the sovereign debt must be r, the investors cost of funds. Investors receive $1/q$ units of consumption for every unit lent. Since the lenders are competitive, expected returns are equalized:

$$1 + r = \frac{1 - E[d']}{q} \tag{2}$$

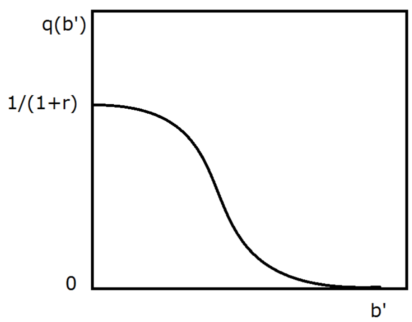
We know from the previous part that the country defaults if $y_2 < -2b_2$. So the probability of default is given by the probability that the endowment falls below this threshold:

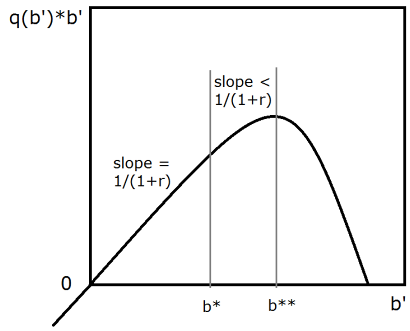
$$\begin{aligned} E[d_2] &= prob(y_2 < -2b_2) \\ &= F(-2b_2) \end{aligned}$$

This gives the desired expression for the bond price. Note that the bond price is a function of borrowing b_2 because the default probability is. Borrowing today is the level of debt tomorrow that the government enters the period with and makes default decisions based on. If output were not i.i.d. but persistent, expectations would be conditional on the current level of output y_1 and thus default probabilities and the bond price would be functions of the current level of output y_1 also.

(c) Use the bond price to explain why and how borrowing is limited endogenously in this model.

Solution:





The bond price reflects default risk. At low debt levels, it is equal to the inverse of the risk free rate, at high debt levels it falls to zero as default risk rises to 1. Bond *revenue* thus displays a Laffer curve -type shape: At low levels of borrowing default risk is zero and thus bond revenue rises with borrowing (the slope of the bond revenue function is the inverse of the risk free rate, for borrowing below b^* in the diagram), while at high levels of borrowing additional borrowing raises revenue by less since default risk is high and the bond price below the risk free rate. The sovereign never borrows to the right of the level marked b^{**} since he can generate the same amount of revenue with lower bond issuance. For borrowing between b^* and b^{**} any debt he issues carries a default premium. He trades off the intertemporal costs and benefits from borrowing - the benefits are revenue generated from bond sales that can be used towards consumption, the costs are repayment (and thus lower consumption, everything else equal) the next period. In the presence of default risk, the benefits diminish as borrowing rises due to default risk, unlike in a standard model without the option to default.

(d) What features are typically added to this basic model to evaluate its quantitative predictions, and why?

Solution:
An infinite horizon and concave utility of course, but the main points are persistent shocks, temporary exclusion from and re-entry to financial markets, some form of output costs. Persistence is added in order to match the income persistence typically faced by countries that we wish to study (very far from i.i.d.). Exclusion and re-entry is also for realism - no re-entry as in the model above makes default more costly, a stochastic probability of re-entry lets us match the empirically observed duration of default episodes in a simple way. Output costs are added as a proxy for the fact that default episodes appear to be associated with poor macroeconomic conditions in the data. Making default relatively more expensive in good times also helps the model match the data: If default is not costly, the model overpredicts default and underpredicts the levels of debt. These are the common additions, although there are other extensions that could also be discussed (haircuts,

endogenous output costs, long-maturity debt).

(e) What is the explanation of the model for the observed pattern of countercyclical interest rates and trade balances experienced by emerging economies?

Solution: In the model as in the data, interest rates tend to be high and the trade balance in surplus in periods when output is below trend. This occurs because of countercyclical default risk - the country faces stronger incentives to default in bad times and in periods of high debt. If shocks are persistent, bad times are associated with subdued borrowing, but relatively high bond prices, as in the data. This is in contrast to the small open endowment economy model without default risk: There, countries optimally save in good times, leading to procyclical trade balances (and of course constant interest rates).